Year 9
Knowledge Organisers

Block: Summer 1
Reasoning with Proportion

- Enlargement & similarity
- Solving ratio & proportion problems
- Rates

YEAR 9 – REASONING WITH GEOMETRY...

Solving ratio & proportion problems

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers
Ratio: a ratio shows the relative size of two variables
Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 → 2 cans of pop = £1.20 (× 0.5)

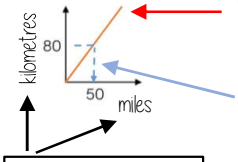
4 cans of pop = £2.40 → 12 cans of pop = £7.20 (× 3)

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running.

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

(÷ 2, × 4 above; × 2, ÷ 4 below)

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	£1.20 ÷ 4	£0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	
	4 cans for £1.20	3 cans for 93p
	4 ÷ £1.20	3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound.

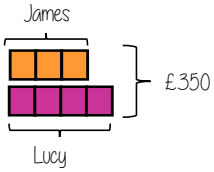
Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

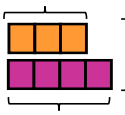
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy
3 : 4 (× 50)
£150 : £200

James = 3 x £50 = £150
Lucy = 4 x £50 = £200



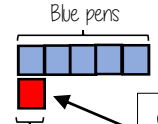
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

□ = one part = 10 pens



One unit = 10 pens

Put back into the question

Blue : Red
5 : 1 (× 10)
50 : 10

Blue pens = 5 x 10 = 50 pens
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9 – REASONING WITH GEOMETRY...

Rates

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

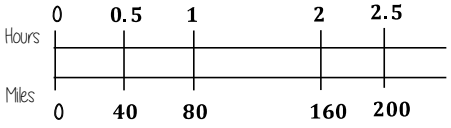
Convert: change
Mass: a measure of how much matter is in an object. Commonly measured by weight.
Origin: the coordinate (0, 0)
Volume: the amount of 3D space a shape takes up
Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

"per" for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

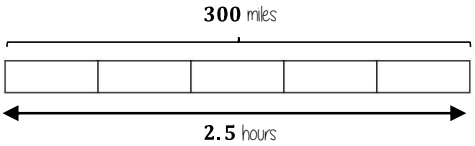
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours
It travels 300 miles.

Bar models can help to calculate mph



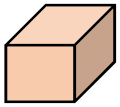
Each part is half an hour
Each part is 60 miles

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



volume of prism = Area of cross section × Depth

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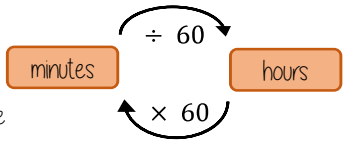
Speed, Distance, Time



Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Substitute in the variables given

$$\text{distance} = \text{speed} \times \text{time}$$

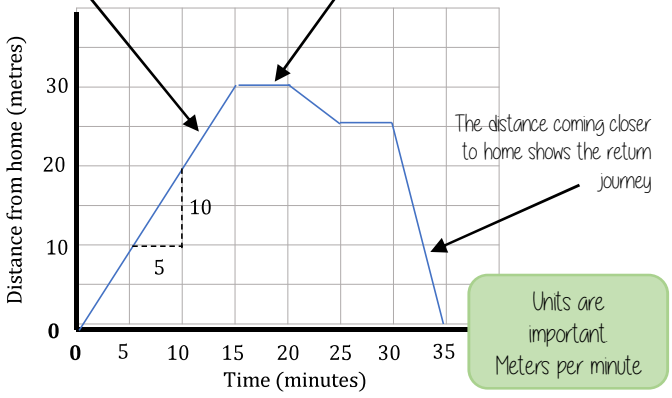
Distance – Time graphs

Gradient = speed

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still



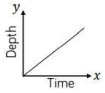
The distance coming closer to home shows the return journey

Units are important
Meters per minute

Flow problems & graphs

This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed

The cylinder will fill at a constant speed



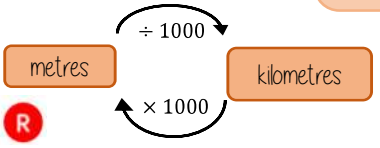
Units are important
Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour
Exchange rates: euros **per** pounds
Density: mass **per** volume



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YEAR 9 – REPRESENTATIONS...

Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

Probability: the chance that something will happen
Relative Frequency: how often something happens divided by the outcomes
Independent: an event that is not effected by any other events.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability – a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.

The probability scale

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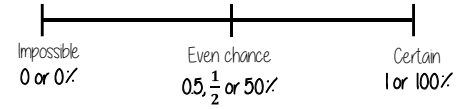
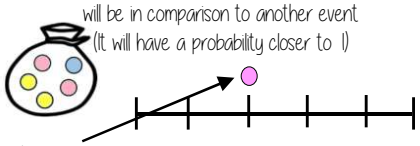


The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

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Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
∴The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(White chocolate) = 1 – 0.15 – 0.35
= 0.5

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times
$0.3 \times 100 = 30$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$0.15 \times 400 = 60$

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1 × Probability of event 2

$P(5) = \frac{1}{6}$ $P(R) = \frac{1}{4}$

Find the probability of getting a 5 and a red

$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables

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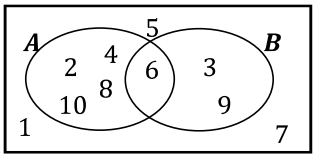

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T