Year 8
Knowledge Organisers

Block: Spring 1
Algebraic techniques

- Brackets, Equations & Inequalities
- Sequences
- Indices

YEAR 8 - ALGEBRAIC TECHNIQUES...

Brackets, Equations & Inequalities

What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms

Substitute: replace a variable with a numerical value

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Product: multiply terms

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Form expressions

For unknown variables, a letter is normally used in its place

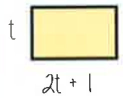
More than – **ADD**

Less than/ difference – **SUBTRACT**

eg 4 more than t ⟶ t + 4

8 less than k ⟶ k - 8

Only similar terms can be grouped together



eg Find the perimeter of this shape.
(Perimeter = length around outside of shape)

t + 2t + 1 + t + 2t + 1 ⟶ 6t + 2

Directed numbers

$+ + \rightarrow +$

$- - \rightarrow +$

$+ - \rightarrow -$

$- + \rightarrow -$

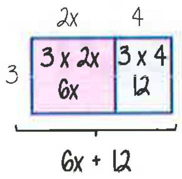
eg a = -5 and b = 2

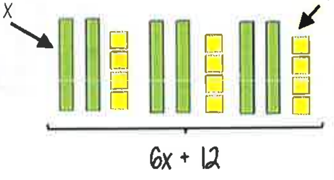
$a^2 = a \times a = -5 \times -5 = \mathbf{25}$

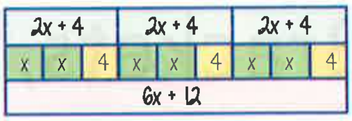
$b + a = 2 + -5 = \mathbf{-3}$

Multiply single brackets

3 (2x + 4)



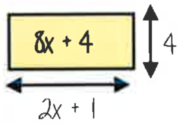




Different representations of 3(2x+4) = 6x + 12

Factorise into a single bracket

8x + 4



Try and make this the **highest common factor**

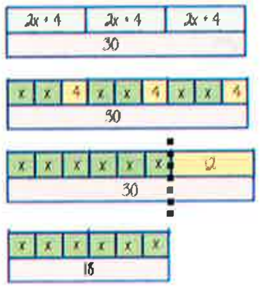
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Solve equations with brackets

3 (2x + 4) = 30



3 (2x + 4) = 30

Expand the brackets

6x + 12 = 30

-12 -12

Substitute to check your answer
This could be negative or a fraction or decimal

6x = 18

÷ 6 ÷ 6

x = 3

Simple inequalities

< less than

≤ Less than or equal to

> More than

≥ More than or equal to

x < 10
Say this out loud
"x is a value less than 10"

10 > x
Say this out loud
"10 is more than the value"

Note
x<10 and 10>x represent the same values

$x + 2 \leq 20$
"my value + 2 is less than or equal to 20"
$x \leq 18$
The biggest the value can be is 18

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$\boxed{3x + 2 > 11}$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

$\boxed{x > 3}$

Check

This would suggest any value bigger than 3 satisfies the statement

3 x 3 + 2 = 11 ✓ 10 x 3 + 2 = 32 ✓

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes ≡

Formula
A rule written with all mathematical symbols
eg area of a rectangle A = b x h

YEAR 8 - ALGEBRAIC TECHNIQUES...

Sequences

What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the nth term
- Recognise geometric sequences and other sequences that arise

Keywords

Sequence: items or numbers put in a pre-decided order
Term: a single number or variable
Position: the place something is located
Linear: the difference between terms increases or decreases (+ or -) by a constant value each time
Non-linear: the difference between terms increases or decreases in different amounts, or by x or ÷
Difference: the gap between two terms
Arithmetic: a sequence where the difference between the terms is constant
Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

Linear and Non Linear Sequences

Linear Sequences – increase by addition or subtraction and the same amount each time

Non-linear Sequences – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

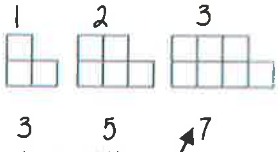
Fibonacci Sequence – look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Sequence in a table and graphically

Position: the place in the sequence



"The **term** in **position** 3 has 7 squares"

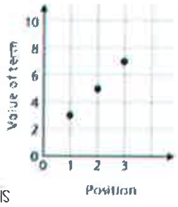
Term: the number or variable
(the number of squares in each image)

In a table

Position	1	2	3
Term	3	5	7

+2 +2

Graphically



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

Sequences from algebraic rules

This is substitution!

$3n + 7$

This will be linear - note the single power of n. The values increase at a constant rate

$3n^2 + 7$

This is not linear as there is a power for n

$2n - 5$ →

Substitute the number of the term you are looking for in place of 'n'

eg
1st term = 2 (1) – 5 = -3
2nd term = 2 (2) – 5 = -1
100th term = 2 (100) – 5 = 195

Checking for a term in a sequence

Form an equation

Is 201 in the sequence 3n – 4?

$$3n - 4 = 201$$

Term to check

Algebraic rule

Solving this will find the position of the term in the sequence
ONLY an integer solution can be in the sequence

Complex algebraic rules

Misconceptions and comparisons

$2n^2$

2 times whatever n squared is

eg
1st term = $2 \times 1^2 = 2$
2st term = $2 \times 2^2 = 8$
100th term = $2 \times 100^2 = 2000$

$(2n)^2$

2 times n then square the answer

eg
1st term = $(2 \times 1)^2 = 4$
2st term = $(2 \times 2)^2 = 16$
100th term = $(2 \times 100)^2 = 40000$

$n(n + 5)$ ←

eg
1st term = $1(1 + 5) = 6$
2st term = $2(2 + 5) = 14$
100th term = $100(100 + 5) = 10500$

You don't need to expand the expression

H Finding the algebraic rule

This is the 4 times table → 4, 8, 12, 16, 20….

4n

↓ ↓ ↓

7, 11, 15, 19, 22 ←

This has the same constant difference – but is 3 more than the original sequence

4n + 3

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 8 - ALGEBRAIC TECHNIQUES...

Indices

What do I need to be able to do?

By the end of this unit you should be able to:

- Add/ Subtract expressions with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices

Keywords

Base: The number that gets multiplied by a power

Power: The exponent – or the number that tells you how many times to use the number in multiplication

Exponent: The power – or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent

Coefficient: The number used to multiply a variable

Simplify: To reduce a power to its lowest term

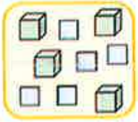
Product: Multiply

Addition/ Subtraction with indices

Coefficient

Power

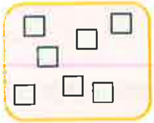
$$\underbrace{\underbrace{5x^2}_{\text{Term}} + \underbrace{4x^4}_{\text{Term}}}_{\text{Expression}} \longrightarrow$$



Each square represents x^2 and each cube represents x^4

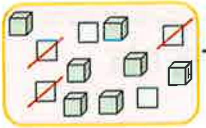
Only similar terms can be simplified If they have different powers, they are unlike terms

$$5x^2 + 2x^2 \longrightarrow$$



$$\longrightarrow 7x^2$$

$$5x^2 + 6x^4 - 3x^2 + x^4 \longrightarrow$$



$$\longrightarrow 2x^2 + 7x^4$$

Divide expressions with indices

$$\frac{24}{36} \longrightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 2 \times \cancel{3}} \longrightarrow \frac{2}{3}$$

$$\frac{5a^3b^2}{15ab^6} \longrightarrow \underbrace{\frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b}} \longrightarrow \frac{a^2}{3b^4}$$

Cross cancelling factors shows cancels the expression

$$\frac{23a^7y^2}{5db^6}$$

This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Multiply expressions with indices

$$\begin{aligned} &4b \times 3a \\ &\equiv 4 \times b \times 3 \times a \\ &\equiv 4 \times 3 \times b \times a \\ &\equiv 12ab \end{aligned}$$

$$\begin{aligned} &5t \times 9t \\ &\equiv 5 \times t \times 9 \times t \\ &\equiv 5 \times 9 \times t \times t \\ &\equiv 45t^2 \end{aligned}$$

$$\left.\begin{aligned} &2b^4 \times 3b^2 \\ &\equiv 2 \times b \times b \times b \times b \times 3 \times b \times b \\ &\equiv 2 \times 3 \times b \times b \times b \times b \times b \times b \\ &\equiv 6b^6 \end{aligned}\right\}$$

There are often misconceptions with this calculation but break down the powers

Addition/ Subtraction laws for indices

$$\underbrace{\begin{aligned} &3^5 \times 3^2 \\ &= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \end{aligned}} \longrightarrow 3^7$$

The base number is all the same so the terms can be simplified

Addition law for indices

$$\boxed{a^m \times a^n = a^{m+n}}$$

$$3^5 \div 3^2 \longrightarrow 3^3$$

$$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \longrightarrow \frac{3^3}{3^0} \longrightarrow \frac{3^3}{1}$$

Subtraction law for indices

$$\boxed{a^m \div a^n = a^{m-n}}$$