Year 10
Knowledge Organisers

Block: Spring 1
Geometry

- Angles and bearings
- Working with circles
- Vectors

St marks
ACADEMY

YEAR 10 – GEOMETRY...

Angles and bearings

What do I need to be able to do?

By the end of this unit you should be able to:

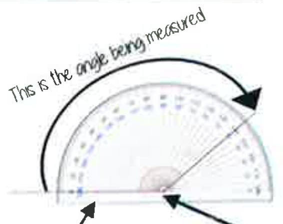
- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

Keywords

Cardinal directions: the directions of North, South, East, West
Angle: the amount of turn between two lines around their common point
Bearing: the angle in degrees measured clockwise from North
Perpendicular: where two lines meet at 90°
Parallel: straight lines always the same distance apart and never touch. They have the same gradient
Clockwise: moving in the direction of the hands on a clock
Construct: to draw accurately using a compass, protractor and or ruler or straight edge
Scale: the ratio of the length of a drawing to the length of the real thing
Protractor: an instrument used in measuring or drawing angles

Measure angles to 180° (R)

This is the angle being measured



Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

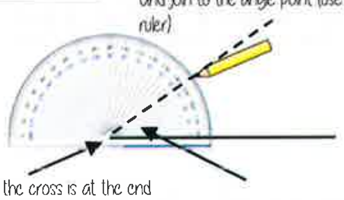
The base line follows the line segment

Make sure the cross is at the point the two lines meet

Draw angles up to 180° (R)

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)



Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation (R)

The letter in the middle is the angle. The arc represents the part of the angle

Angle Notation: three letters $\hat{ABC}$ This is the angle at B = 113°

$\angle ABC$ is also used to represent the angle at B

Scale drawings (R)

1 : 20

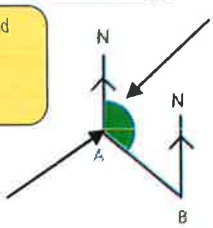
For every 1cm on the model there are 20cm in real life

Remember: Scale drawings ONLY change lengths and distances. Angles remain the same

Understand and represent bearings

- A bearing is always measured from NORTH
- It is always given as three figures

The angle indicated starts from the North line at A and joins the path connecting A to B



This angle shows the bearing of **B from A**

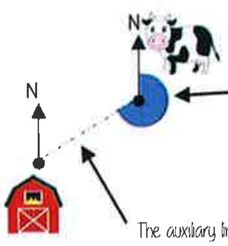
The bearing of B from A is calculated by measuring the highlighted angle

The sentence... "Bearing of ____ from _____" is really important in identifying the bearing being represented

Using **estimation** it is clear this angle is between 090° and 180°

Directions

Measure and read bearings



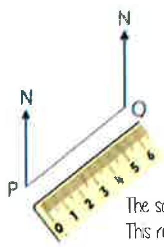
The bearing of the cow to the barn

This angle is measured from NORTH
It is measured in a clockwise direction
Estimation indicates this angle is between 180° and 270°
Use a protractor to measure accurately
Remember bearings are written as three figures

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember – angles DO NOT change size in scaled drawings



The bearing measurements do not change from "real life" to images

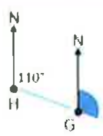
The units in the ratio scale are the same

The scale may need to be calculated from the image. This represents 30km from P to Q

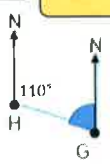
6cm = 30km
6:3,000,000

Bearings with angle rules

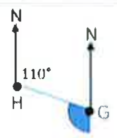
Because two North lines are PARALLEL....



They form **corresponding angles** and therefore are the same size



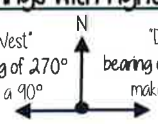
They form **co-interior angles** and add up to 180°



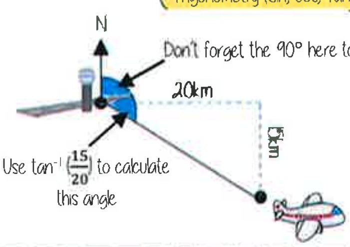
They form **alternate angles** and therefore are the same size

Bearings with right-angled geometry

Look for Right-angles
Pythagoras
Trigonometry (Sin, Cos, Tan)



A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off



Don't forget the 90° here too

Use $\tan^{-1}\left(\frac{15}{20}\right)$ to calculate this angle

YEAR 10 – GEOMETRY...

Working with circles

What do I need to be able to do?

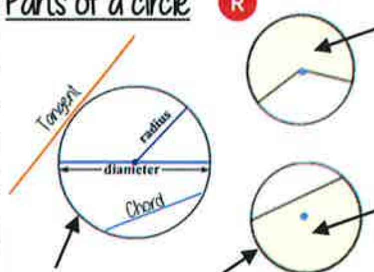
By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere
- Understand and use surface area of a cone, cylinder and sphere

Keywords

Circumference: the length around the outside of the circle – the perimeter
Area: the size of the 2D surface
Diameter: the distance from one side of a circle to another through the centre
Radius: the distance from the centre to the circumference of the circle
Tangent: a straight line that touches the circumference of a circle
Chord: a line segment connecting two points on the curve
Frustum: a pyramid or cone with the top cut off
Hemisphere: half a sphere
Surface area: the total area of the surface of a 3D shape

Parts of a circle R



Sector (part of the circle made from two radii)

Segment (part of the circle made from a chord)

Circumference

An arc is a part of the circumference

Fractional parts of a circle

A circle is made up of 360°

Formula to remember
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$

30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$

$\frac{270}{360}$ of a full circle (in degrees)

$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360}$ × circumference

$= \frac{240}{360} \times 9\pi$

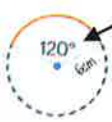
$= \frac{2}{3} \times 9\pi = 6\pi$

Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ × circumference $+ 2r$ $= 6\pi + 9$

Sector area



Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = 12\pi$

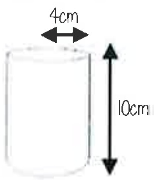
Volume of a cone and a cylinder

Volume Cylinder = $\pi r^2 h$

A cylinder is a prism – cross section is a circle

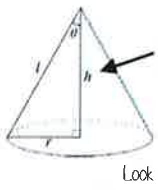
Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base



$V = \pi r^2 h$
$= \pi \times 42 \times 10$
$= \pi \times 160$
$= 160\pi$ cm²

Give your answer in terms of π' means NOT in terms of pi $= 502.7$cm²



The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Volume of a sphere

Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value

Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

$= \frac{4}{3} \times \pi \times 27 = 36\pi$

A hemisphere is half the volume of the overall sphere $= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders

Surface area of a sphere

Surface area = $4\pi r^2$

Radius = 5cm

Surface area = $4\pi r^2$

$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

The curved surface area of a sphere $= 100\pi$

A hemisphere has the curved surface AND a flat circular face

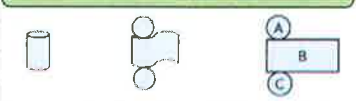
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $= 75\pi$

Surface area of cones and cylinders

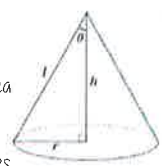
Surface area cylinder = $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles

Curved surface area Cone = πrl



Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

YEAR 10 – GEOMETRY...

Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

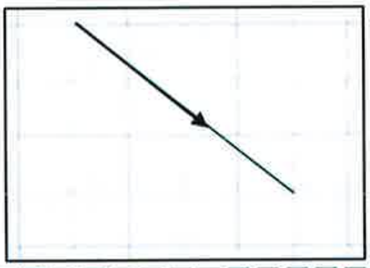
Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

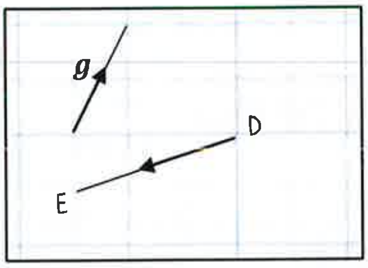
Movement along the x-axis → $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ ← Movement along the y-axis →



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E

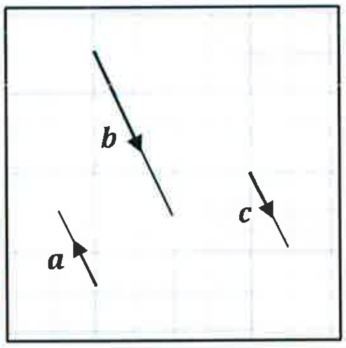
$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction

$\boldsymbol{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

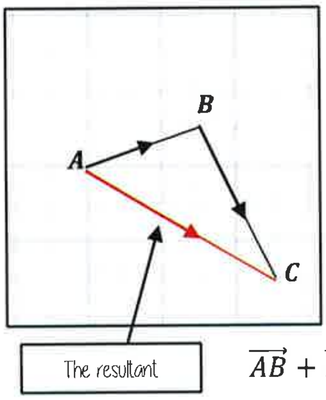
$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$\overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 3+2 \\ 1+-4 \end{pmatrix}$

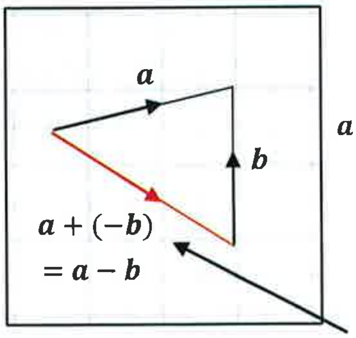
$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$



Look how this addition compares to the vector $\overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Addition and subtraction of vectors



$\boldsymbol{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5 + & -0 \\ 1 + & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1